

Best Times to Trade Stocks using ARIMA and ANN methods

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Abstract: This paper develops a simple methodology to determine the best times to trade stock market prices using both time series and trading rule methods. Firstly a best model is fitted to the observed time series data set relating to closing prices of a single stock using auto-regressive integrated moving averages (ARIMA) and artificial neural network (ANN) approaches. Secondly this paper uses short and long term moving averages to determine a best trading rule over “sell, buy or no trades”. A classification procedure of isolating a best trading rule (BTR) among a set of possible trading strategies is formulated and implemented in three data sets. The performances of ARIMA and ANN methods are compared. The measure of performance is taken to be the expected net-profits that can be realised using each strategy when applied to actual and fitted data sets. ANN is found to provide a better fit to the original series as well identify strategies that lead to higher returns.

Keywords: ANN, ARIMA, Best Trading Rule, Expected net-profits and Fixed Horizon Method.

I. INTRODUCTION

Every prospective stock trader is faced daily with deciding whether to buy new stock, to sell current stock or to do nothing. Experienced traders tend to use a combination of indicators to arrive at a buy/sell/do nothing decision, but less experienced traders may not be able to recognize the appropriate patterns of the market factors using the baskets of indicators available. Sub-optimal decisions lead to low profits and often actual losses. One of the most popular research topics is finding methods that objectively determine the best times to buy, sell or to keep existing stocks based on historical data. Two main competing frameworks exist, one based on time series methods applied to historical data on the performance of stocks, called technical analysis and the other based on evaluating the intrinsic value or current market fundamentals of the actual company (see eg. Fama (1965) and Neftci (1991)).

Although Fama (1965) cast doubts as to whether technical or fundamental analysis can really provide useful information as to when to buy, sell or keep stocks, traders continue to rely on one or other of the two frameworks. Pring (1980) argued that the success of technical analysis relies on investors' beliefs that any changes in trends of financial indices are a result of reactions to changing economic environment including monetary, political and psychological changes. The use of the intersection of a short term moving average series $\{MA_S(t)\}$ with a long-term moving average, $\{MA_L(t)\}$ of the original series $\{X_t\}$ has gained popularity since the work of Neftci (1991) on technical trading methods. Moving averages smooth out noise in the original series without fitting any parametric model and hence point to the general direction of a given series. The degree of smoothness depends on the length of the moving average – i.e. number of time points aggregated to get a given MA value.

Prediction of future states of a random process faces many challenges while dealing with data sets like a time series exhibiting chaotic behaviour. For predicting a one step ahead estimate $\hat{x}(t)$ of a stock price at time ‘t’, it becomes necessary that the past history of the $x(t)$ process be examined with a minimum of error which is $(x(t) - \hat{x}(t))$ as we

did in the above modelling of ARIMA(1,0,0) and that of the NN(1,1,1) fittings. Any Model of ARIMA (p,d,q) family fitted to an observed time series does not yield good predictive power if the underlying mapping is of strongly non-linear type. Hence ANN models have been used to deal with non-linearity if any, as an alternative to ARIMA models since the middle of 1990s. (see Allendew et al. (2002)).

A few performances of ANN and ARIMA methods and a comparison study have been highlighted for a case of stock processes in Adebisi *et al.* (2014). A combined ARIMA (p,d,q) -ANN (i,h,o) model approach has been used by Zhang (2003) to select an optimum model to do predictions for the stock market index EGX30 in Elwasify (2015). Another hybrid ARIMA-ANN model building approach has been used by Zhang (2003) which improves the forecasting accuracy considerably over both ARIMA and ANN results. Since the fitted values to the actual values of the current illustration are all satisfactory values, there is no need to go for a combined ARIMA (p,d,q) - ANN (i,h,o) approach. Those readers who are interested to know about the neural network architecture and training strategy rules are referred to Mueller and Reinhardt (1991).

II. TRADING RULES BASED ON CROSSING OF MOVING AVERAGES

Let $Y_k(t)$ denote the value of a moving average of length 'k' at time t for a financial series $X(t)$. Then $Y_k(t)$ can be expressed as

$$Y_k(t) = \begin{cases} \frac{1}{k} \sum_{j=0}^{k-1} X(t-j); & t \geq k \\ \frac{1}{t} \sum_{j=0}^{t-1} X(t-j); & t < k \end{cases} \quad \dots(1)$$

Here the moving average of length $k > 2$ at time $t > k$ is the simple average of the values of the most recent k including the current value. Unlike in classical time series analysis, there is no need for centring the moving average even for even length. A typical technical trading rule based on moving average method requires that moving average for two different values of k be computed and monitored until the two series meet or cross each other. This is interpreted as a buy or sell signal.

Let $Y_S(t)$ and $Y_L(t)$ denote the value of the short and long term moving average series at time t , respectively. Thus 'S and L' denote the time span of the short and long term moving average series respectively. It is remarked that the longer time spans are less affected by daily price fluctuations than the shorter time span series. When prices fall below the moving average they have the tendency to keep on falling. On the other hand, when prices rise above the moving average they tend to keep on rising. In technical trading, buy and sell signals are activated when the series $\{Y_S(t)\}$ and $\{Y_L(t)\}$ cross each other as follows:

Let our trading rule $G(t)$ be determined by the selection of 'short time span S and long time span L' of the moving average series $Y_S(t)$ and $Y_L(t)$, i.e.

$$G_{SL}(t) = \begin{cases} -1 & \text{signal to sell existing share for a high price} \\ 0 & \text{signal to do nothing/keep} \\ 1 & \text{signal to buy new shares for a low price} \end{cases} \quad (2)$$

Let us now introduce a trading rule explicitly as below:

$$G_{SL}(t) = \begin{cases} -1 & \text{if } Y_S(t) < Y_L(t) \text{ and } Y_S(t-1) \geq Y_L(t-1) \\ 1 & \text{if } Y_S(t) > Y_L(t) \text{ and } Y_S(t-1) \leq Y_L(t-1) \\ 0 & \text{otherwise} \end{cases} \quad \dots(3)$$

Now a question arises on how to make use of these estimates to minimize error if committed with a trading strategy on “buy”, “sell” and “no trading” days. A simple error function called the minimization objective function $e_{SL}(t, h)$ is constructed based on short and long term moving averages.

In the next section we propose now algorithm to determine a best combination of moving average lengths (S^* , L^*) among the possible pairs (S, L) of moving average (MA) functions

$$Y_S(t)=MA_S(t) \text{ and } Y_L(t)=MA_L(t) \quad \dots (4)$$

that minimize objectively defined objective error function and hence has potential to maximise returns on investment.

This prediction algorithm is developed based on the relationship between two different moving averages for three different time series/data sets comprising the (i) actual data series, (ii) series fitted by auto-regression integrated moving averages ARIMA and (iii) fitted series by using neural networks, (each of size 20 for our illustration).

Let the daily data for a stock trading such as the observed close value be $X(t)$ for $t=1, 2, \dots, N$, where N is the current day. Then the h -day future/ahead return on trading at time t is $z(t+h)$ defined by

$$z(t+h) = \left(\frac{X(t+h) - X(t)}{X(t)} \right) \quad \dots(5)$$

Furthermore, we determine the optimal values for short and long MA series that lead to the highest returns for a given trading horizon “ h ”. We use both the traditional ARIMA class of models and neural networks to determine predictive models for rate of returns. The algorithms require that the data set be split into a historical part for training the models, the evaluation part corresponding to the trading horizon considered and the testing period of the same length as the length of the trading horizon. The pair $\langle \text{input, desired output} \rangle$ of values for training the algorithm based on known data for $t=1, 2, \dots, N$ and data of stock prices for testing period called test-data are considered as below:

$$\langle \text{input, output} \rangle = \langle (X(t), X_1(t)): t=1, 2, \dots, (N-2), (N-1), N \rangle \quad \dots(6)$$

where $X_1(t)$ is to be obtained by an appropriate model fitted by either ARIMA or ANN approach. We have, for a given h ,

$$\text{test-data} = \{X(t); t=(N+1), (N+2), \dots, (N+h)\} \quad \dots(7)$$

For given series data $\{X(t); t=1, 2, \dots, (N)\}$, estimates $X_1(t)$ of $X(t)$ are obtained using the ARIMA model that minimises the prediction mean square and has the largest AIC statistic. From the fitted model, estimates are obtained for

$$\bar{z}(t+h) \text{ for } t=1, 2, \dots, N. \quad \dots(8)$$

If estimates in equation (8) are reliable towards maximizing the profit then the trader can use the methodology proposed here for decision making. This method provides optimal solution if the series is linear.

Under the neural network method the optimal number of neurons are trained with the same input data set and tested with the ‘test-data’. We denote the fitted values from the neural network as follows:

$$z^*(t+h) \text{ for } t=1, 2, \dots, N \quad \dots(9)$$

In this paper we suggest best values for ‘S and L’ where “best value” refers to the value that minimises some error function. Suppose the observed closing stock price series $\{X(t)\}$ is traded according to the trading decisions using $G_{SL}(t)$. Let the ‘ h ’ day period’s rate of return (gain/loss) function defined by (5) be denoted as $P_{SL}(t, h)$, i.e.

$$z(t+h) = P_{SL}(t, h) = \left(\frac{X(t+h) - X(t)}{X(t)} \right)$$

Thus $P_{SL}(t, h)$ measures the gain or loss on an amount $X(t+h)$ to be invested after ‘ h ’ time units (days) from trading decision made at the current time t to relate with $G_{SL}(t)$. It is now advisable for further decision making to explore the consequences of making a decision at time ‘ t ’ with regard to profit/loss and thus an error function $e_{SL}(t, h)$:

- if $G_{SL}(t)=1$ (i.e. buy) and $P_{SL}(t,h) < 0$ then it helps the trader to buy a low price stock to make a profit amount of “ $-P_{SL}(t,h)$ ”; decision of $G_{SL}(t)=1$ is correct
- if $G_{SL}(t)=1$ and $P_{SL}(t,h) > 0$ then it forces the trader to buy a high price stock to make a loss amount of “ $P_{SL}(t,h)$ ”; decision of $G_{SL}(t)=1$ is not correct.
- if $G_{SL}(t)=-1$ (i.e. sell) and $P_{SL}(t,h) < 0$ then it helps the trader to sell a low price stock to make a loss amount of “ $-P_{SL}(t,h)$ ”; decision of $G_{SL}(t)=-1$ is not correct.
- if $G_{SL}(t)=-1$ and $P_{SL}(t,h) > 0$ then it helps the trader to sell a high price stock to make a profit amount of “ $P_{SL}(t,h)$ ”; decision of $G_{SL}(t)=1$ is correct.

The error function $e_{SL}(t,h)$ is given as follows:

$$e_{SL}(t,h) = \begin{cases} -z(t+h) & \text{if } G_{SL}(t)=1, z(t+h) < 0 \\ -z(t+h) & \text{if } G_{SL}(t)=1, z(t+h) > 0 \\ z(t+h) & \text{if } G_{SL}(t)=-1, z(t+h) < 0 \\ z(t+h) & \text{if } G_{SL}(t)=-1, z(t+h) > 0 \\ 0 & \text{if } G_{SL}(t)=0 \end{cases} \quad \dots(10)$$

Let β denote the total number of times the ratio $R(t)$ series takes the unity value for $t=h, (h+1), (h+2), \dots, N$. Then total expected rate of return (TERR) is

$$TERR(S, L, h) = \frac{1}{\beta} \sum_{t=h}^N e_{SL}(t,h) \quad \dots(11)$$

As the non-zero values -1 and 1 of $G_{SL}(t)$ alternate over a period of N time points, the trader is able to compute the value of $TERR(S, L, h)$ for various choices of S and L combinations while prefixing the value of h as desired. It is a fact that for each feasible input (S, L, h) , $TERR(S, L, h)$ gives a negative or a positive rate of return.

For example, when $S=2$, then the corresponding moving average series $\{Y_{S=2}(t)\}$ will move as closely as possible to the observed stock prices $\{X(t)\}$ as compared with the movements of the longer period moving averages $Y_{L>3}(t)$. Hence, for $S=2$, and $h=7$, it is advisable to inspect the positive values of the $TERR(S, L, h)$ by varying $L=3, 4, 5$ while tracing the $BTR(S=2, L^*, h=5)$ which is expected to maximize desired profit/rate of return.

This exercise needs a procedure of estimating $P_h(t)$ and $G_{SL}(t)$ functions that maximise the profit $TERR(S, L, h)$ of the trading strategy $BTR(S=2, L^*, h=7)$. Such learning task of finding the estimates $\bar{P}_h(t)$ and $\bar{G}_{SL}(t)$ of $P_h(t)$ and $G_{SL}(t)$ respectively can be done through parametric and nonparametric methods. Parametric types include forecasting method based on past and present data by analysis of trends and time series regression used for modelling and forecasting of economic, financial, and biological systems. One of the better choices of nonparametric methods is to use a general nonlinear model to be implemented by a multi-layer-feed-forward neural network based back propagation algorithm. Data points in the training set are excluded from the test (validation) set.

Trading Algorithm and Illustration:

This section formulates an algorithm for our trading method based on past stock closing price changes $\{X(t)\}$ of a single stock observed at $t=1, 2, \dots, N$. It involves short and long term moving averages $Y_S(t)$ and $Y_L(t)$ to locate the feasible trading positions $G_{SL}(t)$ for buying and selling and no trading. Considering the desired output as $X(t)$ to a set of lagged values $\underline{X}(t-1)=\{x(t-j):j=1,2,\dots,(t-1)\}$ for $t=2, 3, \dots, N$, a neural network with optimal number of hidden layers is

trained to produce the expected/estimated $\bar{x}(t)$. To obtain a BTR(S^* , L^*) policy, neural-estimates $P^*(t,h)$ are computed and together with $G_{SL}(t)$ used in the TERR function. This process is repeated for different feasible combinations of (S , L : $L=(S+1)$, $(S+2)$,..., h). The following illustration explains briefly the different steps involved while moving towards BTR(S^* , L^*):

Step 1: Firstly coca cola time series data from yahoo `ko<-getSymbols("KO", start="2010-01-01", auto.assign=F)` is downloaded and 20 values on the closing prices $X(t)$ ($t=1,2,\dots, 20$) are extracted for this illustration. Fitted model by forecast software using `auto.arima` to the observed X is that of the AR(1) model *i.e.* ARIMA(1,0,0) with non-zero mean 45.8051685 and its standard error 0.2698015 :

$$x(t) = 7.64628 + 0.8330695 x(t-1) + e(t)$$

where $e(t) \sim \tilde{N}(0,1)$ is a white noise process. It actually predicts $x(t)$ one day ahead only.

Step 2: For training data set $\langle \underline{X}(t-1), X(t) \rangle = \langle \text{input, desired output} \rangle$, a feed forward neural net "NN(1,1,1)" is trained with back-propagation algorithm and thus the neural model is fitted. Actual data, fitted values by the ARIMA model and Neural model are reported in the appendix-A.

Step 3: Desired h -days ahead rate of returns $P(t,h=5) = z(t+h)$ for actual data, $\bar{z}(t+h)$ for ARIMA(1,0,0) and $z^*(t+h)$ for the neural NN(1,1,1) model are then computed and reported in the same table of the appendix-A.

Step 4: A graph showing the actual data versus fitted values by ARIMA (1,0,0) and the neural network NN(1,1,1), that is one neuron in the input, one in the hidden and one in the output layers) is drawn. Another graph showing the actual rate of returns $z(t+h)$, and the returns obtained from those fitted values due to ARIMA(1,0,0) and the neural NN(1,1,1) is also drawn in Fig. 1a and Fig. 1b, respectively.

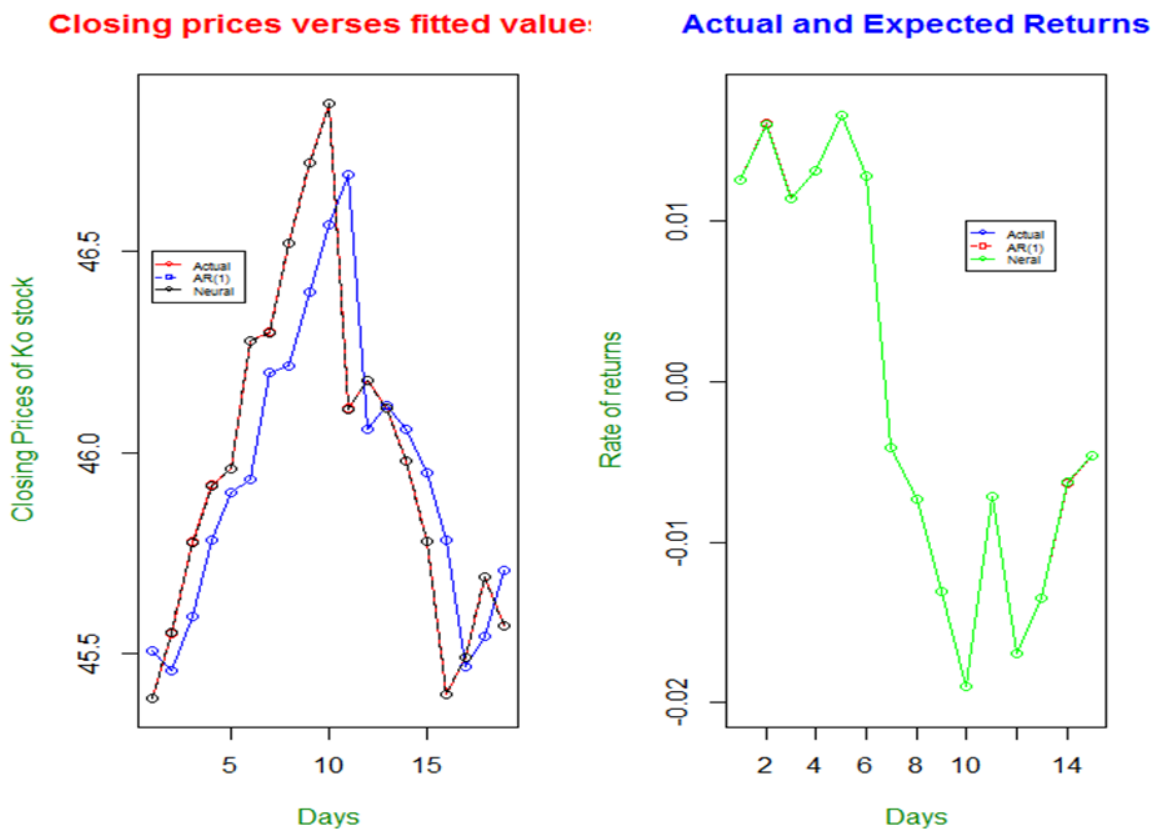


Fig 1: Closing Prices

Fig 2: 5-days' ahead rate of returns

A simple inspection of Fig 1 shows that movements of actual data on the closing prices and fitted values by the NN(1,1,1) coincide with a little variation while that of the ARIMA(1,0,0) values are slightly lower than the actual data. Also inspection of Fig 2 shows that movements of $z(t+h)$, $\bar{z}(t+h)$, and $z^*(t+h)$ move up and down like a single curve.

Step 5: For this example, the function that minimises the $TERR(S=2, L, h=5)$ for $L=3, 4$ and 5 corresponds to $L=3$. Hence a $BTR(S^*=2, L^*=3, h=5)$ is selected using those $z(t+h)$, $\bar{z}(t+h)$ and $z^*(t+h)$ values calculated from the (i) actual, (ii) fitted series by AR(1) and (iii) fitted series by neural NN(1,1,1). Table I provides details for $BTR(S^*=2, L^*=3, h=5)$ inclusive of the outcomes computed on $TERR(S^*=2, L^*=3, h=5)$

Dtat	$N-h-\beta$	β	sum	$TERR(S=2,3, h=5) = \text{sum}/\beta$
Actual	10	4	-0.031370492447	-0.007842623112
AR(1)	11	3	-0.018918730783	-0.006306243594
Neural	10	4	-0.031437578106	-0.007859394526

Here, each β value gives the number of opportunities to trade (“buy”+“sell”) out of the trading period ($N-h=19-5=14$) days, during which “buy” and “sell” signals alternate depending upon the $MA_S(t)$ and $MA_L(t)$ curves crossing each other such that $MA_S(t)$ crosses the $MA_L(t)$ from below produces a “buy” signal and each such crossing from the above produces the “sell” signal.

Graphical Representation of the Trading Rule:

A graphical display of the $BTR(S^*=2, L^*=3, h=5)$ is now illustrated in Fig 3. These types of signals are shown in the following graphical charts from which one could compare the prediction power of ARIMA approach with that of the Neural approach.

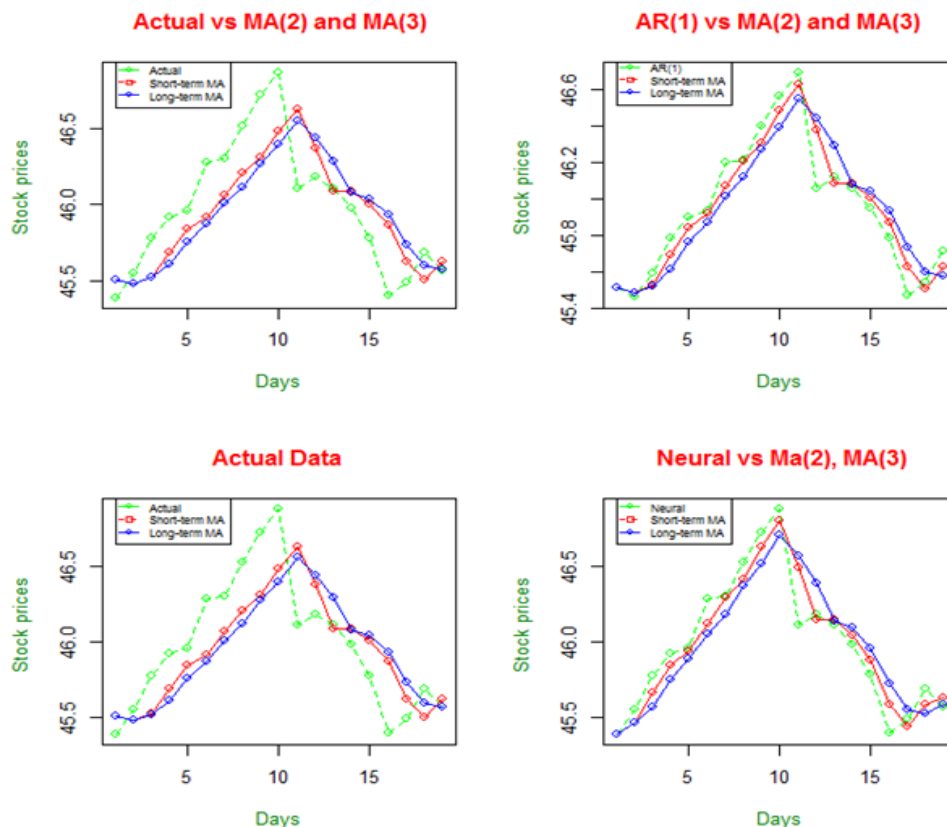


Fig 3: Best Trading Rule based on Moving Averages

The proposed BTR($S^*=2$, $L^*=3$, $h=5$) signals “BUY”, if the short-term MA curve crosses the long-term MA from below. A “SELL” signal is issued when the short-term MA crosses the long-term MA curve from above. Careful inspection of the results of Table I and the graphs enables us to draw a conclusion that the neural method of one-step (ahead) prediction performs better than the ARIMA approach since most of the neural predictions fall very close to the actual values as compared to the corresponding fitted values by the ARIMA method.

Evaluation of prediction Algorithm for $z(t+1)$:

We explore the performance of AR(1) (i.e. ARIMA) and NN(1,1,1) (i.e. ANN) models on $z(t+1)$ instead of $X(t)$, since our primary interest is the comparison of hit rates based on the signs of $z(t+1)$. Prediction accuracy can now be evaluated and compared between the ARIMA and ANN methods using the resulting one day (ahead) rate of returns reported in Table II and TABLE III:

TABLE II: 50% hit rate from ARIMA Model			
Date	Actual series	Forecasted	Accuracy
2017-09-21	-0.008335143581	-0.0001275933641	1
2017-09-22	0.001980416447	-0.0008659644588	0
2017-09-25	0.004386868141	0.0001203760627	1
2017-09-26	-0.002629828413	0.0003344041067	0

TABLE II: Showing 75% hit rate from Neural Model NN(1,1,1)			
Date	Actual series	Forecasted	Accuracy
2017-09-21	-0.008335143581	0.002823348571	0
2017-09-22	0.001980416447	0.004359226654	1
2017-09-25	0.004386868141	0.008335143581	1
2017-09-26	-0.002629828413	-0.001980416447	1

These predicted estimates of rate of returns show that 50% hit rate is observed from the ARIMA model fitting while 75% hit rate is observed from the ANN model fitting.

III. CONCLUSION

The objective of this paper was to determine best times to buy new shares or sell/keep existing stock, based on historical performance of the stock series. ARIMA and ANN frameworks were used to determine the best fitting models to existing stock. Short term and long term moving averages of the fitted values from each model were then computed and the times when the two MA series became equal (crossing times or when their ratio was equal to 1) were determined and used to make trading decisions. The performance of each model was then evaluated using the rate of returns over a fixed time horizon.

The results show that Artificial Neural Network (ANN) method provided the best fits to the original series as well as the best average future returns for the data set that was used. Both the ARIMA and ANN methods were superior to using moving averages purely on the original technical data. The attraction of ANN is that it does not make any parametric assumptions, and yet still outperforms the methods based on time series models that rely on several assumptions.

We recommend that for given technical data, the approach adopted in this paper should be utilised. It is likely that for certain data and time horizons, ARIMA model might be found to be more appropriate. Furthermore, when the ARIMA and ANN give conflicting results, it could point to an inherent property of the stock series that was previously unknown,

APPENDIX - A

Time	Stock	AR(1)	Neural	Time	Stock	AR(1)	Neural
t	KO.Close	Model	Model	t	KO.Close	Model	Model
2017/08/29	45.450	45.646	NA	2017/09/13	46.870	46.567	46.869
2017/08/30	45.390	45.509	45.389	2017/09/14	46.110	46.692	46.109
2017/08/31	45.550	45.459	45.553	2017/09/15	46.180	46.059	46.180
2017/09/01	45.780	45.593	45.777	2017/09/18	46.110	46.117	46.110
2017/09/05	45.920	45.784	45.918	2017/09/19	45.980	46.059	45.980
2017/09/06	45.960	45.901	45.959	2017/09/20	45.780	45.951	45.780
2017/09/07	46.280	45.934	46.279	2017/09/21	45.400	45.784	45.400
2017/09/08	46.300	46.201	46.299	2017/09/22	45.490	45.468	45.490
2017/09/11	46.520	46.217	46.521	2017/09/25	45.690	45.543	45.690
2017/09/12	46.720	46.401	46.721	2017/09/26	45.570	45.709	45.570

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